On the Willingness to Pay to Reduce Risks of Small Losses

Christophe Courbage
The Geneva Association
Geneva, Switzerland
christophe_courbage@genevaassociation.org

Béatrice Rey
ISFA - University Claude Bernard
Lyon, France
beatrice.rey-fournier@univ-lyon1.fr

Abstract

While willingness to pay is a common concept to measure the benefit gained from a reduction in the probability of loss, it is still questionable how it is linked to risk aversion and risk elimination behaviors, and how it is affected by the presence of an exogenous source of risk. By focusing only on risks of small losses, this article sheds light on these issues and provides new results on the determinants of the willingness to pay.

Keywords: Willingness to pay; Small loss; Risk aversion; Background risk

JEL classification: D81

1. Introduction

Willingness to Pay (WTP) makes it possible to evaluate the monetary value one is ready to forgo to benefit from a reduction in the probability of loss (Jones-Lee, 1974). This concept is often used to measure the benefit of prevention or in other cost-benefit analysis where the benefit of an individual or collective decision needs to be evaluated and compared to its cost. A large literature has questioned the determinants of the WTP. One important issue is its link with risk aversion. If empirical studies concur on the existence of a positive relationship between WTP and risk aversion (see e.g. Cropper and Oates, 1992; Smith et al., 2004), this relation does not easily find support on theoretical grounds. Indeed, Eeckhoudt et al. (1997) showed that the WTP to reduce a financial risk is not necessarily increasing in the Arrow-Pratt measure of risk aversion depending on conditions on individual preferences. In the same vein, Jullien et al. (1999) and Chiu (2000) pointed out that only if the initial probability of loss is below a threshold that depends on individual preferences, a more risk averse individual has a higher WTP. A second issue is how the WTP to reduce the probability of loss is linked to the WTPs to eliminate the risk. Indeed, Eeckhoudt et al. (1997) showed that, only in the restrictive case of constant absolute risk aversion (CARA), the WTP for a partial reduction in the probability of loss equals the difference between the WTP for risk elimination in each state. A third important issue is that of the impact of a background risk on the WTP to reduce the first risk. Indeed, economic decision makers often confront other sources of risk that affect their attitudes towards the first risk (see e.g. Gollier and Pratt, 1996; Franke et al., 2006). In addition, this issue brings further information on the potential errors of empirical elicitations in evaluations of the WTP which do not consider the presence of multiple sources of risk, as stressed by Eeckhoudt and Hammitt (2001) in the case of infinitesimal probability reduction.

The aim of this article is to theoretically address these issues under the specific framework of small monetary risks as defined by small losses. Contrary to numerous works (see e.g. Eeckhoudt and Hammitt, 2001; Godfroid, 2001; Bleichrodt and al., 2003), we do not approximate the WTP assuming a small reduction in the probability of loss, but we approximate the WTP considering a small loss in case of damage, whatever the size of the reduction in probability. Indeed, the idea is that large risks are usually covered by insurance, leaving to the charge of individuals the amount of the deductible or of the co-payment. Even in the case of full insurance, the occurrence of the risk has always some negative effects that the individual would like to avoid (administrative cost, psychological cost, etc...). Most of the risks individuals need to prevent against are risks of small loss. By focusing on risks of small loss, we provide new results on the determinants of the WTP. First, we characterise conditions for which the WTP is always increasing in risk aversion. These conditions depends only on the probability distribution of total wealth. Then we link the WTP to reduce the probability of loss to the WTPs to eliminate the risk, without any restriction on the utility function. Finally, we show that the conditions to have an increase in the WTP to reduce one risk in the face of an independent background risk of loss depends on the nature of the background risk, whether it is in the form of a pure damage risk or in the form of any unfair risk. While decreasing absolute risk aversion (DARA) is required in the first case, hypothesis on risk vulnerability, a recent concept introduced by Gollier and Pratt (1996), is required in the second case.

In what follows, section 2 provides conditions for which the WTP is always increasing in the Arrow-Pratt measure of risk aversion. Section 3 deals with the link between the WTP for a partial risk reduction and the WTP for total risk reduction. Section 4 addresses the introduction of an independent background risk of loss, either in the form of pure damage risk or in the form of any unfair risk. Finally, section 5 offers a short conclusion.

2. The model

Consider an individual with an initial wealth level equal to W_0. With probability p_0, an accident occurs and the individual loses a monetary amount L. The expected utility of the individual with a twice differentiable, increasing and concave vNM utility function u writes as:

$$E[U] = (1 - p_0)u(W_0) + p_0 u(W_0 - L) \tag{1}$$

Let's note d the individual's WTP[1] to benefit from a partial reduction of the probability of loss, i.e. to switch from the lottery $\widetilde{L}_0 = [-L, 0; p_0, 1 - p_0]$ to the lottery

[1] As stressed in Chiu (2000), the concept of WTP is equivalent to investigating the optimal choice of self-protection given an assumed relationship between self-protection spending and the loss probability. We will then use interchangeably either one term or the other.

$\widetilde{L}_1 = [-L, 0; p_1, 1-p_1]$, with $0 < p_1 < p_0$; d is given by the following equation:

$$(1-p_0)u(W_0) + p_0 u(W_0 - L) = (1-p_1)u(W_0 - d) + p_1 u(W_0 - L - d) \tag{2}$$

For small values of L, using Taylor series developments, we can approximate each side of equation (2) around L, giving:

$$d(L) \simeq L\Big(\frac{\partial d}{\partial L}\Big)_{L=0} + \frac{L^2}{2}\Big(\frac{\partial^2 d}{\partial L^2}\Big)_{L=0} + o(L^2) \tag{3}$$

since $d(0) = 0$. Differentiating equation (2) with respect to L provides:

$$\frac{\partial d}{\partial L} = \frac{p_0 u'(W_0 - L) - p_1 u'(W_0 - L - d(L))}{(1-p_1)u'(W_0 - d(L)) + p_1 u'(W_0 - L - d(L))} \tag{4}$$

and thus

$$\Big(\frac{\partial d}{\partial L}\Big)_{L=0} = p_0 - p_1. \tag{5}$$

Differentiating equation (4) with respect to L, and using equation (5), we obtain:

$$\Big(\frac{\partial^2 d}{\partial L^2}\Big)_{L=0} = [p_0(1-p_0) - p_1(1-p_1)]\Big(-\frac{u''(W_0)}{u'(W_0)}\Big)$$

Equation (3) is thus equivalent to:

$$d(L) \simeq L(p_0 - p_1) + \frac{L^2}{2}[p_0(1-p_0) - p_1(1-p_1)]A^u(W_0) \tag{6}$$

where $A^u(W_0) = -\frac{u"(W_0)}{u'(W_0)}$ denotes the Arrow-Pratt measure of risk aversion.

There exists a positive relation between the individual's WTP and his/her level of risk aversion as measured by the Arrow-Pratt measure of risk aversion if the term in brackets in equation (6) is positive. It is rather easy to show that it arises in two cases: (i) $\forall$ $p_0 \leq \frac{1}{2}$, and (ii) $\forall p_1 < 1 - p_0$. We then have:

Proposition 1. *For small losses, an increase in the Arrow-Pratt measure of risk aversion increases the WTP to reduce risk if either* (i) $p_0 \leq \frac{1}{2}$ *or* (ii) $p_1 < 1 - p_0$.

Both the initial level of loss probability and the magnitude of the reduction in loss probability play an important role. If $p_0 \leq \frac{1}{2}$ whatever the reduction in the probability of loss, the WTP increases with risk aversion. If $p_0 > \frac{1}{2}$, the magnitude of the reduction in the loss probability needs to be considered. Indeed the condition $p_1 < 1 - p_0$ means that the higher p_0, the lower p_1 is. Hence, for $p_0 > \frac{1}{2}$, the higher the initial loss probability, the more important the reduction in the loss probability is required for a more risk averse individual to increase his WTP. In others words, for a high loss probability ($p_0 > \frac{1}{2}$), the higher the loss probability, the more efficient[2] prevention activity has to be to increase the WTP of a more risk averse individual.

This result departs from other works in the literature. In a seminal paper, Dionne and Eeckhoudt (1985) showed that there always exists a positive relation between the level of self-protection and the degree of risk aversion in the case of quadratic utility functions and an initial probability of loss strictly inferior to $\frac{1}{2}$. Eeckhoudt et al. (1997) discussed that the WTP to reduce a financial risk is not necessarily increasing in the Arrow-Pratt measure of risk aversion depending on conditions on individual preferences. In the same vein, Jullien et al. (1999) showed that the level of probability for which self-protection is increasing with risk aversion depends upon restrictions on the utility function. This was also showed by Chiu (2000) in terms of WTP. Our results are different from theirs in the sense that the level of loss probability we consider is completely exogenous to the individual and does not depend on properties of the utility function.

It is important to note that this result can also be interpreted in terms of variance of the risk. Indeed, equation (6) can be rewritten as:

$$d(L) \simeq L(p_0 - p_1) + \frac{1}{2}[V(\widetilde{L}_0) - V(\widetilde{L}_1)]A^u(W_0) \qquad (7)$$

where $V(\widetilde{L}_i)$ is the variance of the lottery $\widetilde{L}_i$. There exists a positive relation between the individual's WTP and her level of risk aversion as measured by the Arrow-Pratt measure of risk aversion if the variance of the total wealth is reduced by the passage from p_0 to p_1. This result is easily explainable since for small risks (as defined by small losses) the variance is a good measure of the risk (see Pratt, 1964). Thus if the passage from p_0

[2]Efficiency of prevention activities refers to the intensity of prevention in reducing the loss probability.

to p_1 represents a decrease in risk (as measured by a lower variance), a more risk averse individual is willing to pay more to reduce the probability of loss. This gives us:

Corollary. *For small losses, an increase in the Arrow-Pratt measure of risk aversion increases the WTP to reduce the probability of loss if the variance of total wealth is reduced with lower probability.*

3. WTP for partial and total risk reduction

In an important contribution, Eeckhoudt et al. (1997) looked at the properties of the WTP and the links existing between various forms of WTP. In particular, they investigated under which conditions on the utility function the WTP for a partial risk reduction from p_0 to p_1 equals the difference between the WTP for risk elimination from p_0 and the WTP for risk elimination from p_1. They showed that such result was valid only in the restrictive case of constant absolute risk aversion (CARA). In our framework, this relation is always verified. Indeed, it is easy to show that equation (6) can be rewritten as:

$$d(L) \simeq d(L, p_0, 0) - d(L, p_1, 0) \tag{8}$$

where $d(L, p_0, 0)$ denotes the WTP for a total risk reduction from p_0 (i.e. risk elimination) and $d(L, p_0, 0)$ denotes the WTP for a total risk reduction from p_1. This gives us:

Proposition 2. *For small losses, the WTP for a partial reduction in the probability of loss equals the difference between the WTP for risk elimination in each state.*

Contrary to Eeckhoudt et al. (1997), we do not need to impose conditions on the utility function to have such form of separability in the WTP. These results can be useful in many contingent valuation methods to elicit the WTP to reduce the probability of loss. Indeed, instead of knowing how much an individual is ready to pay to reduce the loss probability, it is equivalent to know how much he is ready to pay to eliminate the risk both in the case of p_0 and p_1. Various studies point out that it is easier and more accurate for an individual to evaluate how much he is ready to eliminate a risk than to reduce it (see...). Also, one could infer the value of the WTP to reduce the loss probability from the observation of individual behaviors on insurance markets. Indeed the demand for insurance is driven by the willingness to eliminate risks.

4. Willingness to pay and background risk

It is often the case that decision makers confront several sources of risk. They usually make a decision about one risk whilst simultaneously facing other risks that may interact with the first one. Not taking into consideration this interaction can significantly affect the study of decision making under uncertainty as shown through extensive literature (see e.g. Gollier and Pratt, 1996; Franke et al., 2006).

The objective here is to examine the impact of an independent background risk of loss, $\tilde{x}$, on the individual's WTP to reduce the first risk. We will consider two different types of background risk. We will either assume that the support of $\tilde{x}$ belong to $\mathbb{R}^-$ which naturally implies $E(\tilde{x}) < 0$, or we will consider any unfair risk, i.e. such as $E(\tilde{x}) \leq 0$ but with $\tilde{x}$ not necessarily belonging to $\mathbb{R}^-$.

The individual's WTP in the presence of an independent background risk, denoted d^B, writes as follows:

$$(1-p_0)Eu(W_0+\tilde{x})+p_0Eu(W_0-L+\tilde{x}) = (1-p_1)Eu(W_0+\tilde{x}-d^B)+p_1Eu(W_0-L+\tilde{x}-d^B) \tag{9}$$

or, that is equivalent

$$(1-p_0)z(W_0)+p_0z(W_0-L) = (1-p_1)z(W_0-d^B)+p_1z(W_0-L-d^B) \tag{10}$$

with

$$z(w) = Eu(w+\tilde{x}) \ \forall w;$$

$z(w)$ being the derived utility function as introduced by Kihlstrom et al. (1981) and Nachman (1982). It states that the optimal decision of an agent with utility function u

and facing the independent background risk, $\tilde{x}$, is the same as the one with utility function z and facing no background risk. As earlier, d^B can be approximated by:

$$d^B(L) \simeq L(p_0 - p_1) + \frac{L^2}{2}[p_0(1-p_0) - p_1(1-p_1)]A^z(W_0) \tag{11}$$

with $A^z(W_0) = -\frac{z''(W_0)}{z'(W_0)}$.

Using equation (6), we thus have:

$$d^B(L) - d(L) \simeq \frac{L^2}{2}[p_0(1-p_0) - p_1(1-p_1)](A^z(W_0) - A^u(W_0)) \tag{12}$$

The comparison between d and d^B depends on the comparison between the Arrow-Pratt measures of risk aversion of z and u, and on the sign of the term in brackets. Depending on the support of the background risk, the conditions on the utility function to have equation (12) positive will be different.

If we focus our attention on a pure damage background risk ($\tilde{x}$ defined on $\mathbb{R}^-$), we obtain the following result:

Proposition 3. *For small losses, the introduction of any independent and background risk with support in $\mathbb{R}^-$ increases the WTP to reduce the probability of the other risk if the individual's utility function is DARA and if either (i) $p_0 \leq \frac{1}{2}$ or (ii) $p_1 < 1 - p_0$.*

Proof. Let's note $\phi(w) = \frac{u'(w)}{Eu'(W_0+\widetilde{x})}$ $\forall w$. According to this notation, we have $A^z(W_0) = E[\phi(W_0+\widetilde{x})A^u(W_0+\widetilde{x})] = E(\phi(W_0+\widetilde{x}))E(A^u(W_0+\widetilde{x})) + cov(\phi(W_0+\widetilde{x}), A^u(W_0+\widetilde{x}))$. Under $u(.)$ concave and DARA, $cov(\phi(W_0+\widetilde{x}), A^u(W_0+\widetilde{x})) \geq 0$. Thus, $A^z(W_0) \geq E(\phi(W_0+\widetilde{x}))E(A^u(W_0+\widetilde{x})) = E(A^u(W_0+\widetilde{x}))$. As the support of $\widetilde{x}$ is in $\mathbb{R}^-$ and DARA is assumed, we have $E(A^u(W_0+\widetilde{x})) \geq A^u(W_0)$ $\forall W_0$, giving $A^z(W_0) \geq A^u(W_0)$. □

Both experimental and empirical works support the DARA hypothesis (see e.g. Levy, 1994), and having a loss probability inferior to $\frac{1}{2}$ is a common situation most decision makers face. Hence proposition 2 strongly supports the idea of Eeckhoudt and Hammitt

(2001) that neglecting background risks is likely to lead to substantial error in estimating the WTP to reduce one risk.

As conditions (i) and (ii) are equivalent to $V(\widetilde{L}_0) - V(\widetilde{L}_1) \geq 0$, it is rather easy to interpret proposition 3. The introduction of a background risk with support in $\mathbb{R}^-$ reduces the final wealth, and since the individual is DARA, it makes the individual more risk averse. He is thus ready to pay more to reduce the probability of loss when a decrease in the probability of loss reduces the risk as measured by a lower variance.

If we now consider any kind of unfair risk, i.e. such as $E(\tilde{x}) \leq 0$, we need to take into account the concept of risk vulnerability introduced by Gollier and Pratt (1996). Following these two authors, the utility function u is risk vulnerable if any unfair background risk makes the individual more risk averse with respect to the first risk, i.e. :

$$E(\tilde{x}) \leq 0 \Longrightarrow A^z(w) \geq A^u(w) \ \ \forall w$$

This gives the following proposition:

Proposition 4. *For small losses, the introduction of any independent and unfair background risk increases the WTP to reduce the probability of the other risk if the individual's utility function is risk vulnerable and if* (i) $p_0 \leq \frac{1}{2}$ *or* (ii) $p_1 < 1 - p_0$.

Interpreting proposition 4 in the same way as we did for proposition 3 gives us: under risk vulnerability, adding an independent and unfair risk makes the individual more risk averse. He is thus ready to pay more to reduce the probability of loss when a decrease in the probability reduces the risk as measured by a lower variance.

Finally, note that the DARA condition is weaker than the risk vulnerability one, in the sense that u risk vulnerable implies u DARA. Hence, the conditions on the utility function vary according to the conditions imposed on the support of the background risk to have an increase in the WTP. There exists a sort of duality between the support of the background risk and the utility function. If the support of the background risk is only in

$\mathbb{R}^-$, then any utility function which is DARA will lead to an increase in the WTP. If any unfair risk is considered, then the utility function has to be risk vulnerable.

5. Conclusion

While WTP is a commonly used concept to measure the benefit gained from a reduction in the probability of loss, it is still questionable how it is linked to risk aversion and risk elimination behaviors, and how it is affected by the presence of an exogenous source of risk. This article sheds light on these issues by focusing only on risks of small loss, and provides new insights on the determinants of the WTP. The conditions exhibited above, being very appealing and realistic, offer natural grounds for experimental and empirical investigations.

References

Bleichrodt, H., D. Crainich and L. Eeckhoudt, 2003, Comorbidities and the willingness to pay for health improvements, Journal of Public Economics 87, 2399-2406.

Chiu, W.H., 2000, On the propensity to self-protect, Journal of Risk and Insurance 67, 555-578.

Cropper, M.L. and W.E. Oates, 1992, Environmental economics: a survey, Journal of Economic Litterature 30, 675-740.

Dionne, G. and L. Eeckhoudt, 1985, Self-insurance, self-protection and increased in risk aversion, Economics Letters 17, 39-42.

Eeckhoudt, L., P. Godfroid and C. Gollier, 1997, Willingness to pay, the risk premium and risk aversion, Economics Letters 55, 355-360.

Eeckhoudt, L. and J. Hammitt, 2001, Background risks and the value of a statistical life, Journal of Risk and Uncertainty 23, 261-279.

Franke, G., H. Schlesinger and R. Stapleton, 2006, Multiplicative background risks, Management Science 52, 146-153.

Godfroid P., 2000, L'assurance de marché et le consentement à payer, Revue économique 51, 41-49.

Gollier, C. and J.W. Pratt, 1996, Risk vulnerability and the tempering effect of background risk, Econometrica 64, 1109-1124.

Jones-Lee, M., 1974, The value of changes in the probability of death or injury, Journal of Political Economy 82, 835-849.

Jullien, B., B. Salanié and F. Salanié, 1999, Should more risk averse agents exert more effort, Geneva Papers on Risk and Insurance Theory 24, 19-25.

Kihlstrom, R.E., D. Romer and S. Williams, 1981, Risk aversion with random initial wealth, Econometrica 49, 911-920.

Levy, H., 1994, Absolute and relative risk aversion: an experimental study, Journal of Risk and Uncertainty 8, 289-307

Nachman, D.C., 1982, Preservation of 'more risk averse' under expectations, Journal of Economic Theory 28, 361-368.

Pratt, J.W., 1964, Risk aversion in the small and in the large, Econometrica 32, 122-136.

Smith, V.K., M.F. Evans, H. Kim and D.H. Taylor, 2004, Do the 'near' elderly value mortality risks differently?, The Review of Economics and Statistics 86, 423-429.