

# **MANAGEMENT STRATEGIES IN LIFE INSURANCE: AN EXAMINATION WITH RESPECT TO RISK PRICING AND RISK MEASUREMENT**

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## **ABSTRACT**

The inclusion of management strategies is typically omitted in the fair valuation process of life insurance liabilities, even though the assumption of constant asset volatility or constant annual surplus participation is unrealistic in practice. In this paper, we investigate the impact of different types of management mechanisms on risk-neutral pricing and shortfall risk. In general, these feedback mechanisms affect the contract's payoff and hence directly influence pricing and risk measurement. To isolate the effect of management strategies on shortfall risk, we develop an extension of the fair valuation method and fix relevant pricing quantities. In particular, we calibrate contract parameters that lead to the same market value and same default-value-to-liability ratio. We then compare shortfall probability and expected shortfall and show the substantial impact of different management strategies acting on the asset and liability side.

## **1. INTRODUCTION**

Implicit options in life insurance contracts are of increasing concern in both academia and practitioners' world, especially since Equitable Life had to close to new business in 2000 due to improper hedging of provided options. Financial pricing of insurance liabilities is usually conducted by using risk-neutral valuation. Management mechanisms are often omitted from this process, even though the assumption of constant asset volatility or constant annual surplus participation rates is rather restrictive and unrealistic in practice. The aim of this paper is to fill this gap and examine the impact of various management mechanisms on risk-neutral pricing and risk measurement for fair contracts with the same market value and the same default-value-to-liability ratio. The strategies considered are based on the insurer's degree of solvency, in the sense that the

investment's volatility or the annual surplus participation rate are reduced when solvency is threatened.

There has been much research on fair valuation of embedded options in life insurance contracts, including Bacinello (2001, 2003a, 2003b), Ballotta et al. (2006b), Briys and de Varenne, (1997), Grosen and Jørgensen (2000, 2002), Hansen and Miltersen (2002), Tanskanen and Lukkarinen (2003). A comprehensive review of this literature can be found in Jørgensen (2004). These authors use the appropriate concept of risk-neutral valuation of insurance contracts to price insurance liability risk; however, they do not consider risk measurement or management strategies within this process.

Among the literature that accounts for actuarial aspects as well, Barbarin and Devolder (2005) present a model that combines financial and actuarial approaches for maturity guarantees with terminal surplus participation, similar to the model used by Briys and de Varenne (1997). Boyle and Hardy (1997) compare an actuarial simulation-based approach with a financial option pricing approach for the pricing and reserving of maturity guarantees. Gatzert and Kling (2006) propose a procedure that considers both pricing and risk measurement, which is also feasible for cliquet-style contracts. They compare the objective real-world risk implied by fair contracts with the same market value for different models and thus identify key risk drivers for fair contracts. Kling et al. (2006) use an actuarial approach to analyze the interaction of contract parameters, regulatory parameters, and management decisions, comparing shortfall probabilities for guarantees with annual surplus participation common in the German market.

The implementation of management strategies is another aspect related to fair valuation and generally leads to modified payoffs. General managerial behavior and financial risk management is discussed and presented, for example, in Babbel and Merrill (2005), Cummins et al. (2001), Santomero and Babbel (1997). In the literature on participating life insurance contracts, Berketi (1999) and Berketi and Macdonald (1999) discuss the effect of risk management on payoff structure and insolvency risk by employing a mean-variance framework, and without following a fair valuation approach. Chadburn (1998) conducts the analysis under an objective real-world measure for different scenarios, comparing long- and short-term management strategies, and studies the influence of management strategies on the policyholders' returns and the insurer's solvency. Hence, the articles mentioned so far contrast return and controlling solvency, thus evaluating the relative benefit of the policyholders' returns. Based on the surplus distribution mechanism and decision rules as introduced in Kling et al. (2006), Bauer et

al. (2006) conduct fair valuation of participating life insurance contracts. The decision rules considered are implied by the insurer's reserve quota. Kleinow and Willder (2006) study hedging strategies and calculate fair values for maturity guarantees, where policyholders participate in the insurer's investment portfolio that is subject to the insurer's management decisions.

The purpose of this paper is to extend previous literature by investigating the impact of management mechanisms on risk-neutral pricing and risk measurement. Considering both approaches allows increased insight into the impact of management strategies. The valuation of insurance liability risk is involved in premium calculation and is thus of interest to insurers and policyholders. Risk measurement of the actual real-world risk in addition to the (risk-neutral) pricing of these contracts provides important information for rating agencies, regulatory purposes (e.g., Solvency II in the European Union), investors, and other stakeholders, among others.

The analysis is based on participating life insurance contracts common in the United Kingdom and other European countries, as described by Ballotta et al. (2006b). These contracts feature a guaranteed interest rate and annual and terminal surplus participation. Three types of management strategies are considered that are based on the insurer's degree of solvency, in the sense that the investment's volatility or the annual surplus participation rate are reduced when solvency is threatened. Two of them regulate the asset side through adjusting the investment's volatility; one strategy controls the annual surplus participation rate, thus acting on the liability side of the balance sheet. These feedback mechanisms affect payoffs as well as pricing and risk measurement quantities. To isolate the effect on shortfall risk, we extend the fair valuation method and fix relevant pricing quantities. In particular, we calibrate contract parameters that lead to the same market value and the same safety level for the insurer. We use the default-value-to-liability ratio as a measure of safety level. We then compare shortfall probability and expected shortfall for the different management strategies.

The remainder of the paper is organized as follows. In Section 2, the model and valuation framework is introduced. In Section 3, we develop a procedure to extend the concept of fair valuation by simultaneously taking the insurer's safety level into consideration and then measuring shortfall risk associated with the contracts. The integration and effect of management strategies on risk given the standard fair valuation approach is described in Section 4. Section 5 examines the influence of management strategies on risk for fair contracts when fixing market value and safety level. The results are then

compared to the results based on the standard fair valuation approach. Section 6 concludes.

## 2. MODEL FRAMEWORK AND FAIR VALUATION OF INSURANCE COMPANY'S LIABILITIES

This section describes the model framework for a life insurance company. The liability structure described is implied by participating life insurance contracts based on the framework suggested by Ballotta et al. (2006b). In particular, the liability structure is described by a guaranteed interest rate, annual surplus participation, and participation in the terminal surplus.

At inception of the contract, the policyholders pay an exogenously given up-front premium  $P_0 = k \cdot A_0$ . The company's initial equity is denoted by  $E_0 = (1 - k) \cdot A_0$ . Hence, the coefficient  $k$  can be considered the leverage of the company. The sum of the initial contributions  $A_0 = E_0 + P_0$  is then invested in assets, the so-called reference portfolio. Under the objective measure  $\mathbb{P}$ , the total market value of assets  $A$  evolves according to a geometric Brownian motion,<sup>1</sup>

$$dA(t) = \mu A(t) dt + \sigma A(t) dW^{\mathbb{P}}(t), \quad (1)$$

with asset drift  $\mu$ , volatility  $\sigma$ , and a  $\mathbb{P}$ -Brownian motion  $W^{\mathbb{P}}$ , where we assume a complete, perfect, and frictionless market. In this setting, solution of the stochastic differential Equation (1) is given by (see, e.g., Björk (2004))

$$A(t) = A(t-1) \exp\left(\mu - \sigma^2 / 2 + \sigma (W^{\mathbb{P}}(t) - W^{\mathbb{P}}(t-1))\right). \quad (2)$$

By changing the measure to the risk-neutral unique equivalent martingale measure  $\mathbb{Q}$ , the drift changes to the risk-free interest rate  $r$ , and the solution of the stochastic differential equation under  $\mathbb{Q}$  is then obtained analogously to Equation (2):

$$A(t) = A(t-1) \exp\left(r - \sigma^2 / 2 + \sigma (W^{\mathbb{Q}}(t) - W^{\mathbb{Q}}(t-1))\right),$$

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<sup>1</sup> Thus  $(W_t)$ ,  $0 \leq t \leq T$  stands for a standard Brownian motion on a probability space  $(\Omega, \mathbb{F}, \mathbb{P})$  and  $(\mathbb{F}_t)$ ,  $0 \leq t \leq T$  is the filtration generated by the Brownian motion.

where  $W^Q$  is a Q -Brownian motion.

$P$  denotes the policyholders' account, the book value of the policy reserves. For any  $t = 1, 2, \dots, T$ , the policy reserves  $P$  annually earn some rate of interest  $r_p(t)$  that includes a guaranteed interest rate and some surplus, which usually depends on the insurance company's financial situation, particularly on the development of its assets. The policy reserve considered here annually earns the greater of the guaranteed interest rate or a fraction  $\alpha$  of the annual surplus generated by the investment portfolio, a so-called cliquet-style guarantee. A contract with  $\alpha = 0$  as in, for example, Briys and de Varenne (1997), is called a point-to-point guarantee. The development of the policy reserve is thus described by

$$P(t) = P(t-1) \cdot (1 + r_p(t)) = P(t-1) \cdot \left( 1 + \max \left[ g, \alpha \left( \frac{A(t)}{A(t-1)} - 1 \right) \right] \right), \quad (3)$$

where  $P(0) = P_0$ . At maturity, a fraction  $\delta$ , denoting the terminal surplus participation coefficient, of the terminal surplus  $B(T)$  is distributed to the policyholders according to the leverage coefficient  $k$  such that

$$B(T) = \max(k \cdot A(T) - P(T), 0).$$

If the company is not able to pay the policyholders' claims  $P(T)$  at maturity, the policyholders receive the total value of the reference portfolio, whereas the equityholders receive nothing. Hence, the default put option can be defined as follows:

$$D(T) = \max(P(T) - A(T), 0). \quad (4)$$

The total payoff to the policyholders  $L(T)$  is<sup>2</sup>

$$L(T) = P(T) + \delta B(T) - D(T).$$

An evaluation of  $L(T)$  using risk-neutral valuation leads to

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<sup>2</sup> In this setting, early surrenders or deaths are ignored as only financial risk is considered. Under the assumption that mortality risk is diversifiable, it can be dealt with using expected values when writing a sufficiently large number of similar contracts.

$$\begin{aligned}
\Pi^* &= E^Q(e^{-rT}L(T)) \\
&= E^Q(e^{-rT}[P(T) + \delta B(T)]) - E^Q(e^{-rT}D(T)) \\
&= \Pi - \Pi^{DPO}.
\end{aligned} \tag{5}$$

Equation (5) shows that the value of the policyholders' claim consists of two parts. The first term corresponds to a (default-free) premium. The second term is the value of the default put option  $\Pi^{DPO}$ , which reduces the default-free premium  $\Pi$  to  $\Pi^*$ . Hence, in valuation of liabilities using option pricing theory, the insurer's possible insolvency is explicitly taken into account by pricing the default put option for the policyholders. In the following, this process is referred to as "risk pricing." Closed-form solutions for the expressions in Equation (5) cannot be obtained for  $\Pi^{DPO}$  due to path dependency;  $\Pi$  can be derived by using Black-Scholes pricing formula, as is done, for example, in Bacinello (2001) and Ballotta et al. (2006a). Monte Carlo simulation is used for all analyses.

The value of the default put option is affected by several parameters. In particular, the leverage coefficient  $k$  determines how much initial debt is incorporated. Thus, the higher  $k$  is, the less initial equity is available and thus the higher the risk of default. In contrast, terminal surplus participation does not induce additional risk for the insurer as the distribution is optional and depends on the value of assets at maturity (see Equation (4)). Hence, shortfall risk is induced only by the guaranteed interest rate and the annual surplus participation.

The equityholders' claim is residually determined as the difference between market value of the reference portfolio  $A(T)$  and the policyholders' claim  $L(T)$ :

$$E(T) = A(T) - L(T) = \max(A(T) - P(T), 0) - \delta B(T).$$

The value of the claim is given by

$$\Pi^E = E^Q(e^{-rT}E(T)). \tag{6}$$

### 3. FAIR VALUATION OF LIABILITIES GIVEN A FIXED SAFETY LEVEL

Even when contracts have the same value, they generally do not exhibit the same risk. In light of this, a procedure is developed in this section to fix relevant quantities within the fair valuation process to ensure comparability. We extend the process of fair valuation by fixing the company's safety level as a second pricing variable. Furthermore, for these contracts, we calculate the associated objective real-world shortfall risk measured with lower partial moments.

In a no-arbitrage setting, an up-front premium is called “fair” if it equals the present value of the liabilities under the risk-neutral measure at time  $t = 0$ . This is expressed as

$$\Pi^* = P_0. \quad (7)$$

Equation (7) is equivalent to the requirement that the value of the equityholders' payoff be equal to their initial contribution, that is,  $\Pi^E = E_0$ . All other combinations would imply arbitrage opportunities (see Equations (5) and (6)) (Ballotta et al. (2006b)). To analyze the effect of fair valuation on the contract parameters, one can calibrate the contract parameters guaranteed interest rate  $g$ , annual surplus participation  $\alpha$ , and terminal surplus participation  $\delta$  such that the equilibrium condition of Equation (7) is satisfied.<sup>3</sup>

Since fair contracts generally do not have the same shortfall risk or exhibit the same safety level, a risk measure must be chosen. A common measure for an insurance company's safety level is the default-value-to-liability ratio  $d$  (see, e.g., Butsic (1994); Barth (2000)). This ratio is defined as the default put option value divided by the value of the liabilities, and thus enables a comparison of insurance companies of different sizes. Conveniently, the default-value-to-liability ratio can be directly derived from the valuation of the liabilities in Equation (5):

$$d = \frac{\Pi^{DPO}}{\Pi}. \quad (8)$$

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<sup>3</sup> See Ballotta et al. (2006b) for an analysis of feasible sets of individual contract parameters when employing fair valuation.

We now require that the contract parameters have a fixed safety level<sup>4</sup>  $d^*$  and simultaneously satisfy the equilibrium condition of Equation (7). This procedure is an extension of the fair valuation process conducted by Ballotta et al. (2006b). The necessary conditions for the procedure to hold are derived as follows. Given Equations (5), (7), and (8), as well as the requirement to fix a safety level  $d^*$  in the sense that

$$d = \frac{\Pi^{DPO}}{\Pi} = d^*,$$

holds, we can derive the two conditions—

$$\Pi^{DPO}(g, \alpha) = P_0 \cdot \frac{d^*}{1 - d^*} \quad (9)$$

and

$$\Pi(g, \alpha, \delta) = P_0 \cdot \frac{1}{1 - d^*}. \quad (10)$$

When Conditions (9) and (10) are both satisfied, risk pricing is fixed in two ways: the contracts are fair ( $\Pi^* = \Pi - \Pi^{DPO} = P_0$ ) and they have the same safety level ( $\Pi^{DPO} / \Pi = d^*$ ). Thus, the procedure is directly derived from the pricing process and provides a higher degree of comparability.

The first step in finding contracts that satisfy these requirements is that the guaranteed interest rate  $g$  and surplus participation  $\alpha$  are calibrated to satisfy Condition (9) for a given safety level  $d^*$ . Crucial in this process is the fact that  $\delta$  has no impact on the value of the default put option  $\Pi^{DPO}$ , as can also be seen in Equation (4). This means that the procedure proposed in this section is not feasible for contracts that do not feature at least one parameter that has no influence on default. For these types of contracts one can proceed as do Gatzert and Kling (2006) to assess the risk associated with fair contracts. Next, given  $g$  and  $\alpha$  found in the first step, the terminal surplus participation coefficient  $\delta$  is adjusted such that Condition (10) holds. For the calibration procedure, the Newton method is implemented. Until otherwise stated, the parameters given are:

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<sup>4</sup> In the case of property-liability insurance, fixed safety levels have been considered by, e.g., Phillips et al. (1998) and Gründl and Schmeiser (2002, 2006).

$$d^* = 9\%, T = 15, \sigma = 15\%, \mu = 9\%, P_0 = 100, E_0 = 10.$$

In addition to the parameter combinations with the same safety level derived with financial pricing, we conduct risk measurement by calculating the corresponding insolvency risk at maturity measured by lower partial moments of degree 0 and 1 with the aim of providing additional information. Even though risk-neutral valuation provides appropriate prices for insurance liability risk, it cannot replace or deliver information about the actual real-world probability or extent of a shortfall. For risk management purposes, policyholders, and other stakeholders, this information should provide substantial additional value. Shortfall probability (under the objective measure  $P$ ) is given by

$$SP = P(A(T) < P(T)),$$

and unconditional expected shortfall<sup>5</sup> can be defined as

$$ES = E^P(\max[P(T) - A(T), 0]).$$

We consider only European option-style contracts and interpret risk solely as possible shortfall at maturity, thereby allowing for negative reserves during the term of the contract, as is done in, for example, Grosen and Jørgensen (2000). Lower partial moments belong to the class of downside risk measures and take only the lower part of the density function into account, which is of main interest to investors and policyholders. Thereby, shortfall probability corresponds to lower partial moment of degree 0 and simply counts the number of shortfall events. Expected shortfall is a lower partial moment of degree 1 and additionally takes the extent of the shortfall into account. Feasible sets of parameter combinations for fair contracts are set out in Table 1. These combinations serve to demonstrate central effects and will be used for all analysis throughout this paper.

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<sup>5</sup> See, e.g., Wirch and Hardy (1999).

**TABLE 1:** Shortfall risk of fair contracts with fixed safety level  $d^* = 9\%$ .

$g$	$\alpha$	$\delta$	<i>Risk pricing</i>				<i>Risk measurement</i>	
			$\Pi^*$	$\Pi$	$\Pi^{DPO}$	$d^*$	SP	ES
0.00%	42.13%	86.29%	100.00	109.89	9.89	9.00%	8.66%	2.31
0.50%	37.84%	87.88%	100.00	109.89	9.89	9.00%	8.13%	2.21
1.00%	33.16%	89.30%	100.00	109.89	9.89	9.00%	7.64%	2.11
1.50%	27.89%	90.58%	100.00	109.89	9.89	9.00%	7.14%	2.02
2.00%	21.53%	91.76%	100.00	109.89	9.89	9.00%	6.66%	1.93
2.50%	11.27%	92.97%	100.00	109.89	9.89	9.00%	6.15%	1.84
3.00%	-	-	-	-	-	-	-	-

Notes:  $g$  = guaranteed interest rate,  $\alpha$  = annual surplus participation,  $\delta$  = terminal surplus participation,  $\Pi^*$  = premium,  $\Pi$  = value of liabilities,  $\Pi^{DPO}$  = value of default put option,  $d^*$  = default-value-to-liability ratio, SP = shortfall probability, ES = expected shortfall.

For all contracts displayed in Table 1,  $\Pi$  and  $\Pi^{DPO}$  are constant and satisfy Conditions (9) and (10). Hence, two risk prices are fixed since the contracts have the same market value ( $\Pi^* = 100 = P_0$ ) and lead to the same safety level  $d^* = 9.00\%$  for the insurance company. The annual surplus participation coefficient  $\alpha$  decreases when the guaranteed interest rate is raised from 0.00% to 2.50% so as to ensure the fairness condition. For  $g = 3.00\%$ , no value of  $\alpha$  can be found such that Condition (9) holds. In addition to risk pricing, risk measurement is conducted by calculating shortfall probability and expected shortfall under the real-world measure.

The terminal surplus participation coefficient  $\delta$  is found in the second step and calibrated to make the contract fair. In the case considered in Table 1, with increasing guaranteed interest rate  $g$ , annual surplus participation rate  $\alpha$  needs to be lowered significantly to ensure a fixed default put option value  $\Pi^{DPO}$ , which leads to a strong reduction in the liability value  $\Pi$ . To ensure that Condition (10) is met, terminal surplus participation rate  $\delta$  has to be raised to keep the contract fair. At the same time, results from risk measurement show that shortfall probability and expected shortfall are decreasing. This result is important as it shows that higher guaranteed interest rates may be offered if the remaining contract parameters are adequately adjusted. Hence, in the example considered, adjusting the parameters to satisfy a certain safety level leads to lower risk as  $g$  is raised, while  $\alpha$  decreases and  $\delta$  increases.

The procedure developed in this section is an extended fair valuation approach that considers the risk associated with fair contracts in two ways. First, the process enables a comparison of contracts with the same market value and the same safety level, measured with the default-value-to-liability ratio. It further extends the financial approach by comparing the real-world shortfall risk associated with these contracts. A different approach is to fix shortfall probability, expected shortfall, or value at risk, instead of fixing the safety level.

#### **4. THE INFLUENCE OF MANAGEMENT STRATEGIES ON PRICING AND RISK MEASUREMENT**

This section introduces the inclusion of management mechanisms in the fair valuation process. Usually, the volatility of assets is reduced when a company's reserves come under pressure. The model thus includes several feedback mechanisms from the insurer's degree of solvency to the asset volatility and to the participation rate. Management mechanisms generally affect the claims' structure. We focus on an analysis of the impact of different strategies on the fair valuation procedure to study how feasible sets of parameters change. In this section, the standard fair valuation approach used in Section 2 is employed, and the results are later compared to the extended pricing approach developed in Section 3.

Implementing these strategies is not related to a certain objective function of insurers, for example, maximization of shareholder value.<sup>6</sup> The aim of this section is discover to what extent such management strategies are beneficial in the sense that shortfall risk at maturity is reduced while still providing fair conditions to policyholders.

In general, one can distinguish between management strategies that control the asset side of the balance sheet (e.g., asset allocation) and strategies that control the liability side (e.g., surplus distribution mechanisms). Several long- and short-term strategies are considered in Chadburn (1998). Management's decision on when and how to intervene also depends on how solvency is measured during the term of the contract, among other things. There are various ways of measuring solvency. One way is to estimate the risk-bearing capital as a measure of solvency, which requires a best estimate of future liabilities (Ballotta et al. (2006a)). Another possibility is to use book values from the balance sheet to determine the solvency level, which is simple and easily accom-

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<sup>6</sup> See Cummins et al. (2001) for an empirical investigation of the general incentive of insurance companies to conduct corporate risk management.

plished. During the term of the contract, solvency can be measured with the so-called statutory equity  $E(t)$ , which is the difference between the market value of assets at time  $t$  and the book value of the liabilities  $P(t)$ . This can be interpreted as the company's reserves.

$$E(t) = A(t) - P(t).$$

For the purpose of this analysis, using statutory equity as an indicator for solvency is appropriate for illustrating the effect of management decisions when they are integrated into the fair valuation process. Based on this solvency measure, three different solvency-driven management strategies are considered: two investment strategies and one strategy that attempts to control liability by controlling the annual surplus participation rate. With respect to the valuation, it is necessary that the insurer's management strategies are predetermined at  $t = 0$ .

To analyze the effect of management strategies in the context of life insurance contracts, we start with fair contracts and a fixed safety level, as given in Table 1. Management intervention changes the claims' structure and the risk of shortfall associated with the contract. To avoid "unfair" contract conditions and to ensure that contracts remain comparable, we successively adjust the contract parameters  $(g, \alpha, \delta)$  to keep the contract fair at inception. To assess the benefits of incorporating risk management on the reduction of shortfall risk, shortfall probability and expected shortfall are provided as in the previous section.

### *Strategy (A)*

The first management strategy is a dynamic solvency-driven investment strategy where the insurance company reduces the investment portfolio's volatility once to  $\sigma_s$  when solvency is threatened, in the sense that

$$\sigma_s = \sigma - 0.05, \quad \text{if } E(t) < x^A \cdot E_0. \quad (11)$$

In this scenario, the investment portfolio's volatility, once reduced, is never again increased, that is, once the volatility is reduced, it remains at that level until maturity, regardless of better economic conditions. The threshold  $x^A$  is chosen appropriately. Certainly, this strategy will affect the policyholders' payoff. Management intervention

can prevent the insurance company from experiencing a shortfall at maturity, which, for the policyholder, can come at the cost of a lower guaranteed maturity payoff  $P(T)$ .

### *Strategy (B)*

Next, Strategy (A) is extended as follows:

$$\sigma_s = \begin{cases} \sigma - 0.05, & E(t) < x_l^B \cdot E_0 \\ \sigma, & x_l^B \cdot E_0 \leq E(t) \leq x_u^B \cdot E_0 \\ \sigma + 0.05, & E(t) > x_u^B \cdot E_0 \end{cases} \quad (12)$$

As for Strategy (A), volatility is reduced when the company's reserves come under pressure, measured with the lower threshold  $x_l^B \cdot E_0$ . However, when the insurer's financial situation eases, portfolio weights are shifted to reach the original volatility. Further, the insurer even raises the volatility of assets to higher than the original level when reserves are high ( $> x_u^B \cdot E_0$ ), thus creating a riskier portfolio. This strategy is more aggressive, as in good times, the reference portfolio's volatility is increased above the original volatility  $\sigma$ .

### *Strategy (C)*

To contrast and compare the results for Strategies (A) and (B), which act on the asset side, a second type of management strategy is considered that regulates the liability side of the balance sheet by controlling the annual surplus distribution. In insurance practice, the participation rate can be changed due to requirements imposed by regulators, or due to a managerial process within the company. At time  $t = 0$ , the insurance company promises the policyholders a guaranteed participation rate  $\alpha_0$  in the annual surplus generated by the reference portfolio. However, this rate can be subject to the insurer's degree of solvency as it could be reduced to zero when solvency is threatened and is only reimbursed when the financial situation recovers to a level above a security buffer ( $x_u^C \cdot E_0$ ):

$$\alpha(t) = \begin{cases} 0, & E(t) < x_l^C \cdot E_0 \\ \alpha_0, & E(t) > x_u^C \cdot E_0 \end{cases}, t = 1, \dots, T. \quad (13)$$

When statutory equity  $E(t)$  is in the range  $[x_l^C \cdot E_0, x_u^C \cdot E_0]$ , the policy reserves are compounded only with the guaranteed interest rate  $g$ , that is,  $r_p(t) = g$  in Equation (3). Analogously to the previous strategies, statutory equity at time  $t$  is used to determine the degree of solvency. In this setting, the policyholder does not receive the missed surplus later, thus accepting a lower guaranteed maturity payoff. Numerical results for Strategies (A), (B), and (C) are given in Table 2 for a guaranteed interest rate of 2.50%. In the analysis of risk pricing and risk measurement, we set  $\mu = 7\%$  (8%, 9%, 10%) for  $\sigma = 5\%$  (10%, 15%, 20%). Furthermore, until otherwise stated,  $x^A = x_l^B = x_l^C = 0.75$ ,  $x_u^B = 1.25$ , and  $x_u^C = 1.15$ .

**TABLE 2:** Influence of Strategies (A), (B), and (C) on fair valuation and shortfall risk; values at time  $t = 0$ .

				Risk pricing				Risk measurement	
	$g$	$\alpha$	$\delta$	$\Pi^*$	$\Pi$	$\Pi^{DPO}$	$d^*$	SP	ES
T 1	1.50%	27.89%	90.58%	100.00	109.89	9.89	9.00%	7.14%	2.02
(A)	1.50%	27.89%	90.58%	99.68	103.90	4.23	4.07%	2.55%	0.39
(A)	1.80%	27.89%	90.58%	100.00	104.94	4.94	4.71%	3.01%	0.49
(A)	1.50%	30.62%	90.58%	100.00	104.61	4.61	4.41%	2.93%	0.46
(A)	1.50%	27.89%	91.91%	100.00	104.23	4.23	4.06%	2.55%	0.39
(B)	1.50%	27.89%	90.58%	100.84	108.27	7.43	6.86%	11.27%	1.95
(B)	0.52%	27.89%	90.58%	100.00	105.21	5.21	4.95%	8.56%	1.26
(B)	1.50%	21.69%	90.58%	100.00	106.34	6.34	5.96%	9.10%	1.46
(B)	1.50%	27.89%	87.19%	100.00	107.43	7.43	6.92%	11.27%	1.95
(C)	1.50%	27.89%	90.58%	99.02	105.63	6.61	6.26%	4.56%	1.07
(C)	2.46%	27.89%	90.58%	100.00	110.26	10.26	9.31%	6.93%	2.03
(C)	1.50%	41.20%	90.58%	100.00	107.46	7.46	6.95%	5.88%	1.41
(C)	1.50%	27.89%	93.95%	100.00	106.61	6.61	6.20%	4.56%	1.07

Notes: T 1 = contract in Table 1 without risk management strategy; (A), (B), (C) = contract in Table 1 with Strategy (A), (B), (C);  $g$  = guaranteed interest rate;  $\alpha$  = annual surplus participation;  $\delta$  = terminal surplus participation;  $\Pi^*$  = premium;  $\Pi$  = value of liabilities;  $\Pi^{DPO}$  = value of default put option;  $d^*$  = default-value-to-liability ratio; SP = shortfall probability; ES = expected shortfall.

Integration of management mechanisms has a significant impact on pricing, safety level, and risk measurement. The first row in Table 2 contains the contract in Table 1 (T 1) for  $g = 1.50\%$ . When the original parameter combinations ( $g, \alpha, \delta$ ) are kept and Strategy (A) is implemented (second row in Table 2), the default-value-to-liability ratio ( $d^*$ ) and shortfall risk (SP, ES) are reduced. However, at the same time, the value of the contract  $\Pi^* = \$99.68$  is lower than the up-front premium \$100 paid by the policyholders. Hence, for policyholders the reduction of shortfall risk through Strategy (A) can come at the cost of a lower expected maturity payoff, a situation implying unfair contract conditions for policyholders as Condition (7) is not satisfied. To overcome a possible conflict of objectives (reduction of shortfall risk vs. fair payoff to policyholders), we successively recalibrate the contract parameters  $g, \alpha$ , and  $\delta$  in order to ensure that the fairness Condition (7) holds.

In this case, as shown in Table 2, the company can offer higher guarantees when Strategy (A) or (C) is implemented. For example, the first part in Table 2 for Strategy (A) illustrates that the insurer can raise  $g$  from 1.50% to 1.80%, increase  $\alpha$  from 27.89% to 30.62%, or, alternatively, raise  $\delta$  from 90.58% to 91.91%, while keeping the market value of the claims equal to  $\Pi^* = \$100$ . Thereby, the contract components  $\Pi$  and  $\Pi^{DPO}$ , safety level, and shortfall risk differ, depending on which parameter ( $g, \alpha, \delta$ ) is adjusted. Overall, even after adjusting the parameters to higher guarantees, risk is lower compared to the situation where no management strategies are applied.

A comparison of the effects of guaranteed interest rate  $g$  and annual surplus participation rate  $\alpha$  on shortfall risk shows that increasing  $g$  leads to a higher risk. Thus, the lowest default-value-to-liability ratio and shortfall risk are obtained when  $\delta$  is adjusted, all other parameters left unchanged.

In contrast to  $g$  and  $\alpha$ , increasing the terminal surplus participation parameter  $\delta$  in case of Strategies (A) and (C) results in a constant shortfall risk since  $\delta$  has no impact on shortfall. Further, it even leads to a marginal decrease in the default-value-to-liability ratio  $d^*$  (e.g., for Strategy (C),  $\delta = 90.58\%$  leads to  $d^* = 6.26\%$  and  $\delta = 93.95\%$  results in  $d^* = 6.20\%$ ). This is because the default put option  $\Pi^{DPO}$  remains constant when raising  $\delta$  (in the previous example,  $\Pi^{DPO} = \$6.61$ ). On the other hand, the value of the liabilities ( $\Pi$ ), ceteris paribus, increases in  $\delta$  (from \$105.63 to \$106.61; see also Equation (5)), which in total leads to a decrease in the default-value-to-liability ratio  $\Pi^{DPO}/\Pi$ . This result is significant as it shows that increasing the terminal surplus par-

participation rate in order to make the contract fair—given this predetermined management strategy—is a way to lower the default-value-to-liability ratio.

Implementing Strategy (B) brings about different results than those obtained by implementing Strategy (A). Since during economic “good” times, the volatility is increased above the original volatility level, Strategy (B) leads to an increase in market value. For  $g = 1.50\%$ , the market value of claims is \$100.84, and thus higher than the up-front \$100 policyholder premium payment. Thus, to make the contracts comparable, guarantees must be reduced to obtain a fair situation. When considering the effect of reducing different parameters, we find that lowering  $g$  to obtain fair contracts leads to the highest risk reduction compared to  $\alpha$  and  $\delta$  for all risk measures considered. Strategy (B)’s effect on risk is interesting. The default-value-to-liability ratio and the expected shortfall are overall lower than in the case where no strategy is integrated, whereas the shortfall probability increases. Hence, given Strategy (B), there are more shortfalls, but they are less severe. This observation also illustrates potential differences between risk-neutral pricing and real-world risk measurement and the dependence of the results on the choice of the risk measure.

The results for Strategy (C) are displayed in the last section of Table 2, and are similar to those of Strategy (A). In particular, it can be seen that reducing the annual surplus participation rate in times of low solvency leads to a lower market value of claims. However, in contrast to Strategy (A), adjusting the guaranteed interest rate from  $g = 1.50\%$  to  $2.46\%$  leads to a default-value-to-liability ratio of  $9.31\%$ , which exceeds the value in the case where no management strategies are applied ( $9.00\%$ ) and also shows a slightly higher expected shortfall value (2.03 compared to 2.02). Shortfall probability is lower than in the case without intervention. In contrast, the adjustment of annual and terminal surplus participation leads to risk reduction. Further analyses revealed that the results are not very sensitive to changes in the thresholds  $x_l^C$  or  $x_u^C$  (see Equation (13)). To sum up, we found that an insurer’s management strategy has a significant impact on the pricing and risk measurement of life insurance liabilities. Starting with fair contracts and a fixed safety level, integrating management strategies into the fair valuation process requires adjustment of the contract specification to ensure at least a certain degree of comparability. Risk differs depending on how the adjustment is accomplished. The results thus depend on the particular management strategy. Overall, risk management aiming to reduce shortfall risk, such as Strategy (A) or (C), allows the insurer to find fair contracts with higher guarantees (by adjusting one parameter) and lower shortfall risk, both at the same time. In contrast, for the more aggressive Strategy (B), an

increase of volatility above the original level requires a decrease of the guarantees, which, in the example considered, still leads to a higher shortfall probability.

However, Table 2 also demonstrated that for the standard fair valuation approach, comparability of contracts is diminished when management strategies are introduced, even when contract parameters are recalibrated to ensure the fairness condition. All quantities in risk pricing and risk measurement differ except for the contract's market value  $\Pi^*$ . To increase comparability among contracts with respect to risk pricing, the next section applies the extended valuation procedure to additionally fix the company's safety level when management strategies are implemented and analyzes the effect of this procedure on risk measurement.

## **5. THE INFLUENCE OF MANAGEMENT STRATEGIES ON RISK MEASUREMENT GIVEN A FIXED SAFETY LEVEL**

The aim of this section is to integrate the approaches and procedures presented in Sections 3 and 4. Thus, given any predetermined management strategy, that is, Strategies (A), (B), or (C), we apply the fair valuation technique developed in Section 3 and find fair contracts with the same market value and the same safety level (see Equations (9) and (10)). This ensures a higher degree of comparability between the contracts as all relevant quantities in risk pricing are fixed and insurers remain in the same risk class. However, at the same time, both  $\alpha$  and  $\delta$  need to be adjusted, which will also have an effect on risk measurement. Results for Strategy (A) (see Equation (11)) are contained in Table 3.

**TABLE 3:** Fair valuation given Strategy (A), fixed safety level  $d^* = 9\%$ , and shortfall risk.

			<i>Risk pricing</i>				<i>Risk measurement</i>	
$g$	$\alpha$	$\delta$	$\Pi^*$	$\Pi$	$\Pi^{DPO}$	$d^*$	SP	ES
0.00%	66.04%	21.35%	100.00	109.89	9.89	9.00%	13.54%	2.75
0.50%	61.95%	36.98%	100.00	109.89	9.89	9.00%	12.04%	2.48
1.00%	57.48%	49.31%	100.00	109.89	9.89	9.00%	10.78%	2.23
1.50%	52.51%	59.02%	100.00	109.89	9.89	9.00%	9.61%	2.01
2.00%	46.88%	66.85%	100.00	109.89	9.89	9.00%	8.56%	1.82
2.50%	40.25%	73.38%	100.00	109.89	9.89	9.00%	7.54%	1.64
3.00%	31.71%	79.01%	100.00	109.89	9.89	9.00%	6.58%	1.47

Notes:  $g$  = guaranteed interest rate,  $\alpha$  = annual surplus participation,  $\delta$  = terminal surplus participation,  $\Pi^*$  = premium,  $\Pi$  = value of liabilities,  $\Pi^{DPO}$  = value of default put option,  $d^*$  = default-value-to-liability ratio, SP = shortfall probability, ES = expected shortfall.

Fixing the quantities in risk pricing enables an isolated consideration of the effect of management strategies on risk measurement. The pattern observable in Table 3 is similar to the one observed in Table 1. Table 3 illustrates that the inclusion of management strategies in the pricing process can allow the company to offer higher guaranteed annual surplus participation rates compared to Table 1. However, the terminal surplus participation rate is significantly lower. As in Table 1, shortfall risk decreases with an increasing guaranteed interest rate, independent of the particular risk measure (SP, ES). The overall risk level, however, depends on the risk measure. Shortfall probability is generally higher than in the case where no management strategies are considered, despite the integration of management mechanisms. Expected shortfall, on the other hand, is lower for guaranteed interest rates greater than or equal to 1.50%. As an example, take  $g = 2.50\%$ : in the case without management strategies (Table 1),  $\alpha = 11.27\%$  and  $\delta = 92.97\%$  correspond to a shortfall probability of 6.15% and an expected shortfall value of 1.84. Given Strategy (A) (Table 3), fair contract specifications require a much higher guaranteed annual participation rate of  $\alpha = 40.25\%$  and a lower terminal participation of  $\delta = 73.38\%$ . Despite the fact that both contracts lead to the same risk pricing values, the corresponding shortfall probability of Strategy (A) is higher (7.54%), whereas the expected shortfall is lower (1.64). Hence, for guaranteed interest rates greater than or equal to 1.50%, Strategy (A) leads to more shortfalls, but these are of

lesser extent, on average. The results for Strategies (B) and (C) (see Equations (12) and (13)) are displayed in Tables 4 and 5.

**TABLE 4:** Fair valuation given Strategy (B), fixed safety level  $d^* = 9\%$ , and shortfall risk.

$g$	$\alpha$	$\delta$	Risk pricing				Risk measurement	
			$\Pi^*$	$\Pi$	$\Pi^{DPO}$	$d^*$	SP	ES
0.00%	53.12%	60.39%	100.00	109.89	9.89	9.00%	21.42%	4.49
0.50%	48.68%	67.15%	100.00	109.89	9.89	9.00%	19.80%	4.05
1.00%	43.77%	72.96%	100.00	109.89	9.89	9.00%	18.03%	3.66
1.50%	38.24%	77.76%	100.00	109.89	9.89	9.00%	16.49%	3.29
2.00%	31.77%	81.80%	100.00	109.89	9.89	9.00%	14.81%	2.96
2.50%	23.12%	85.58%	100.00	109.89	9.89	9.00%	13.12%	2.61
3.00%	-	-	-	-	-	-	-	-

Notes:  $g$  = guaranteed interest rate,  $\alpha$  = annual surplus participation,  $\delta$  = terminal surplus participation,  $\Pi^*$  = premium,  $\Pi$  = value of liabilities,  $\Pi^{DPO}$  = value of default put option,  $d^*$  = default-value-to-liability ratio, SP = shortfall probability, ES = expected shortfall.

**TABLE 5:** Fair valuation given Strategy (C), fixed safety level  $d^* = 9\%$ , and shortfall risk.

$g$	$\alpha$	$\delta$	Risk pricing				Risk measurement	
			$\Pi^*$	$\Pi$	$\Pi^{DPO}$	$d^*$	SP	ES
0.00%	111.84%	-55.98%	100.00	109.89	9.89	9.00%	24.69%	9.72
0.50%	94.80%	10.02%	100.00	109.89	9.89	9.00%	19.50%	7.19
1.00%	82.07%	43.99%	100.00	109.89	9.89	9.00%	15.15%	4.70
1.50%	66.66%	71.19%	100.00	109.89	9.89	9.00%	11.21%	3.12
2.00%	47.55%	85.10%	100.00	109.89	9.89	9.00%	8.31%	2.25
2.50%	18.67%	92.34%	100.00	109.89	9.89	9.00%	6.29%	1.86
3.00%	-	-	-	-	-	-	-	-

Notes:  $g$  = guaranteed interest rate,  $\alpha$  = annual surplus participation,  $\delta$  = terminal surplus participation,  $\Pi^*$  = premium,  $\Pi$  = value of liabilities,  $\Pi^{DPO}$  = value of default put option,  $d^*$  = default-value-to-liability ratio, SP = shortfall probability, ES = expected shortfall.

For Strategy (B), the annual surplus participation rate is lower compared to the case in Table 3. This is due to the higher volatility, which implies higher chances of participation in the insurer's reference portfolio and also higher risk. Overall, for both strategies, the risk level is higher compared to the case where no strategies are implemented, even though all contracts have the same risk pricing quantities. This observation is independent of the choice of the risk measure (SP, ES). As observed previously, shortfall risk generally decreases with increasing guaranteed interest rates. For Strategy (C) (see Equation (13)) in Table 5, when  $g = 0.00\%$ ,  $\delta$  has to be negative, in theory, to ensure fairness between policyholders and equityholders. These results illustrate that the implementation of various management strategies does not necessarily lead to a reduction of shortfall risk, even when contracts keep the same risk pricing quantities.

## 6. SUMMARY

Management strategies are usually not taken into account in the fair valuation process of life insurance liabilities, even though the assumption of constant asset volatility or constant annual surplus participation rates is restrictive. In this paper, we investigated the effect of different types of management strategies on pricing and risk measurement. To isolate the influence on shortfall risk, we extended the fair valuation method and fixed market value and insurer safety level. Numerical results were illustrated by means of a participating life insurance contract.

For contracts found with the extended pricing approach, risk was calculated under the objective real-world measure using lower partial moments. One main finding was that this pricing procedure leads to a lower risk when the guaranteed interest rate is raised, while the annual surplus participation coefficient decreases and terminal surplus participation rate increases.

Taking the previous analysis as a starting point, the model was made more realistic in a next step by including three types of dynamic solvency-driven management strategies. These strategies are feedback mechanisms from the insurer's degree of solvency to asset volatility and the annual surplus participation rate. First, the effect of management strategies was studied for the standard valuation procedure by recalibrating the contract parameters one at a time (guaranteed interest rate, annual and terminal surplus participation rate) to obtain fair contracts with the same market value.

Our numerical analysis showed that results depend on which parameter is adjusted and on the choice of management strategy. We observed that management strategies aiming to reduce shortfall risk by reducing volatility or the annual surplus participation rate require higher guarantees to obtain fair contracts and still yield a lower shortfall risk. In contrast, the strategy of increasing volatility above the original level during times of good solvency requires a decrease of guarantees. This mechanism led to a higher shortfall probability, but to a lower expected shortfall, and hence to more numerous shortfalls of low severity. The identification of key risk drivers further depend on whether the risk measure is derived during the risk-neutral valuation process (default-value-to-liability ratio) or under the objective measure (shortfall probability and expected shortfall).

To increase comparability among contracts with respect to risk pricing, the developed extended pricing procedure was applied to contracts with and without management strategies. The results show that the integration of management strategies requires higher annual surplus participation and lower terminal surplus participation for a given guaranteed interest rate in order to keep market value and safety level equal to the contracts without intervention. We found that even though these contracts have the same risk pricing quantities, the integration of management strategies can imply a higher shortfall risk.

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