An Equilibrium Analysis of an Insurance Market with Horizontal Differentiation

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Abstract

This article analyzes an insurance market with horizontal differentiation and brand loyalties using a spatial competition model. We build a three-stage game that includes sales promotion (the first stage), varieties (the second stage), and premiums (the third stage). The following results are found. First, the insurance firm with higher brand loyalty realizes a greater degree of differentiation and sets higher premiums than rival firms. This potentially explains why domestic insurance firms that possess relatively higher brand loyalty realize a greater degree of differentiation and set higher premiums than foreign insurance firms. Second, introducing variety regulation leads to a lower degree of differentiation and level of sales promotion. Such regulation may be desirable in terms of social surplus. This finding can explain why variety regulation in domestic insurance is more rigid than in business insurance. Because there are many individuals with brand loyalty in the domestic insurance market, the disadvantages stemming from the introduction of variety regulation appear smaller.

Keywords

Insurance, Horizontal differentiation, Brand loyalty, Regulation

JEL Classification

G22, L12, L50

1. Introduction

After the insurance business law changed in Japan in 1996, insurance regulations were relaxed, especially after 1998.[1] For example, life (non-life) insurance firms could enter the non-life (life) insurance market by establishing a subsidiary. A further example is that the regulation of premiums and insurance products were relaxed. These deregulations encouraged not only price competition in the Japanese insurance market,

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[1] At least some studies have focused on the deregulation of Japanese insurance market after the changes in the insurance business law. For example, see Kwon and Skipper (1997), Hayakawa et al. (2000), Pope (2004), and Pope and Ma (2005).

but also non-price competition, such as the development of new insurance products, increased sales promotion, and so on.[2]

Following deregulation, foreign insurance firms entered the market, setting relatively low premiums in order to attract new individuals. In contrast, Japanese insurance firms did not wish to lower their premiums, focusing instead on the improvement of insurance products and after-sales service. Why did these strategies differ? Furthermore, although deregulation occurred, some regulations still exist, especially in the domestic insurance market. Are these regulations then socially desirable?

The purpose of this article is to analyze the price and non-price competition in the insurance market in order to consider these questions. More particularly, a spatial competition model (horizontal differentiation model) first discussed by Hotelling (1929) is used. However, this analysis includes two factors in the original spatial competition model as follows. First, there are some individuals who always purchase insurance products from a specific insurance firm, regardless of conditions such as the level of premiums. Second, in order to induce the individuals to purchase insurance, insurance firms can undertake sales promotion activities.

The remainder of this article is as follows. Section 2 briefly explains the spatial competition model. Section 3 constructs a three-stage model including sales promotion activities, varieties, and premiums as strategic variables. Concluding remarks are made in Section 4.

2. A brief explanation of the spatial competition model

Before the change in the insurance business law, insurance firms always needed to obtain approval from the regulator if they wanted to sell new insurance products, ostensibly to maintain stability in the market. As a result, the time lag between the development and sale of new products was long and insurance firms sold few consumer-oriented insurance products. In fact, before deregulation there was little difference among insurance products. And even if some differences existed, individuals could not generally recognize them.[3]

After the change in the insurance business law, each insurance firm was encouraged to develop new insurance products, especially in the automobile insurance market. Hence, in order to analyze the actual insurance market, we build a model that includes not only price competition but also non-price competition. Even if the price is higher than that of rivals, some insurance products may be popular because of advantages from non-price aspects. Thus, insurance firms consider how to design their insurance products and differentiate them from rivals.

According to the theory of product differentiation, product differentiation has both advantages and disadvantages. One advantage is the avoidance of fierce competition from rivals. If the degree of product differentiation is high, individuals may recognize that one product is different from another. Firms can then set higher prices. The disadvantage is that this lowers the market share because each highly differentiated

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[2] Hereafter, ‘price’ and ‘premium’ are used interchangeably.

[3] One item of evidence is that individuals compare insurance products less than other ordinary products. For example, see Johnson-O’Conner et al. (1984). In addition, McDowell (1989) insists that even if there were different insurance products, courts have frequently said that they are same.

product is not usually desired by as many individuals as a more 'standard' product would be.

In turn, product differentiation can be divided into two types, 'vertical differentiation' and 'horizontal differentiation'. This classification depends on how we differentiate the products. The former is the situation where the products are differentiated by quality. Quality is defined as product characteristics where all individuals have same preference order. In contrast, the latter is the situation where the products are differentiated by variety. Variety is defined as product characteristics where the preference order differs among individuals. Taking cars as an example, the abilities of a car (maximum speed, fuel consumption) represent quality, while the color of the car is an example of variety.

In the insurance market, we can show product differentiation as follows. First, consider vertical differentiation. What is quality in the insurance market? Because insurance products are intangible and many individuals cannot understand what the insurance clauses mean, the contents of an insurance contract are unlikely to represent quality. Instead, the level of after-sales service may be quality because all individuals prefer to receive a high level of after-sales service, especially following accidents. Thus, if there are no income constraints, all individuals purchase high-quality (a high level of after-sales service) insurance products. However, each individual has a different evaluation of these services. Some individuals wish to receive high-level services, even if they need to pay higher premiums, while others desire to pay lower premiums, even if they will not be able to receive sufficient services in the case of an accident. Given this situation, it is easy to imagine that some insurance firms will sell 'low-price, low-quality' insurance products while others will sell 'high-price, high-quality' insurance products.

Next, consider horizontal differentiation. A characteristic of horizontal differentiation is that each individual has a different preference order for insurance products. Thus, even if the premiums are the same, every individual purchases different insurance products in accordance with their ideal variety points. One example of variety in the insurance market is the composition ratio of death and living protection. Individuals who expect a high (low) death rate prefer to purchase highly death-protected (living-protected) insurance products. Given this situation, some insurance firms sell 'highly death-protected' insurance products while others sell 'highly living-protected' insurance products.

Of course, both product differentiations are mixed in the actual insurance market. However, this analysis is limited to the discussion of horizontal differentiation.[4] The reason for this limitation is because of the expansion in recent years of individuals' needs in relation to insurance products. That can be interpreted as diversification on the evaluation axis. It also complicates the set of the individuals' preference order. Given these complexities, each insurance firm considers how to differentiate its insurance products. In other words, every insurance firm decides the sales position and attracts the individuals.

[4] For a discussion of vertical differentiation in the insurance market, see Schlesinger and Schulenburg (1993), Gravelle (1999), and Okura (2005).

There are some earlier studies of horizontal differentiation in the insurance market. For example, Schlesinger and Schulenburg (1991) analyze the switching behavior of individuals who have some ideal varieties. Tsutsui et al. (2000) discuss the premium–dividend scheme competition using a horizontal differentiation model. Jack (2006) utilizes the horizontal differentiation model in order to shed light on optimal risk adjustment in the insurance market under asymmetric information.

In contrast, our model relaxes the (implicit) assumption that all individuals choose insurance firms in accordance with price and variety. In reality, there are some individuals who decide upon an insurance firm regardless of price and variety. Instead, they decide upon an insurance firm according to its brand name and reputation because the insurance products appear to have higher brand loyalty than other ordinary goods. In this situation, every insurance firm engages in sales promotion in order to enhance their own brand loyalty. Thus, we need to build a spatial competition model that includes not only price and variety but also brand loyalty.

3. Horizontal differentiation in the insurance market

3.1. The model

There are two insurance firms: A, B. We set out the following three-stage game. For the purpose of simplicity, in the first stage, insurance firm B only decides the level of sales promotion. Assume that the purpose of this sales promotion is to enhance firm B's brand and reputation. In the second stage, after observing the level of sales promotion, both insurance firms simultaneously decide upon the variety of insurance product. In the third stage, and given the decisions in the first and second stages, both insurance firms simultaneously decide their premium.

There is justification for this three-stage setup in relation to the order of decision-making.[5] According to Tirole (1988), variables that can be changed more easily in the short run are chosen in the later stage when a multiple-stage game is modeled. Changing the price level is much easier than changing the variety because the regulation of variety is more rigid than the regulation of premiums. Also, changing the variety is much easier than changing the level of sales promotion because sales promotions can be viewed as a long-term investment.

Assume that all individuals choose either the purchase of one unit of insurance product from either insurance firm or no purchase at all. Their utility is assumed to be separable in variety and premium. Thus, their utility function is:[6]

$$U_i = u_i - p_i - d^2 \tag{1}$$

where u_i represents the reservation price of the individual toward the insurance product sold by insurance firm $i \in \{A, B\}$, p_i denotes the premium of insurance firm *i*, and d

[5] In relation to the following, see also Okura (2007).

[6] The reason why the disutility forms a quadratic function is that the existence of the equilibrium in this game (variety–price game) has already been confirmed by D'Aspremont et al. (1979) and Neven (1985).

indicates the distance between the variety that the individual actually purchased and the ideal variety. Thus, d^2 represents the disutility due to the mismatch of varieties.

Moreover, suppose that four types of individuals exist in the insurance market as follows.

Type 1: They always purchase the insurance product from insurance firm A regardless of premiums and varieties because they prefer the brand or reputation of insurance firm A.

Type 2: They always purchase the insurance product from insurance firm B regardless of premiums and varieties because they prefer the brand or reputation of insurance firm B.

Type 3: They purchase insurance products from the more desirable (that with less disutility) insurance firm.

Type 4: They never purchase the insurance product regardless of premiums and varieties.

The type each individual belongs to depends on the reservation prices u_A and u_B. If u_A is sufficiently large relative to u_B, this individual belongs to type 1. If u_B is sufficiently large relative to u_A, this individual belongs to type 2. If u_A and u_B are not small and the difference between u_A and u_B is small, this individual belongs to type 3. If both u_A and u_B are small, this individual belongs to type 4. We denote N as the total number of individuals. Also, the number of type 1 to 3 individuals represent N_A, N_B, and N_N, respectively (thus, the number of type 4 individuals represents $N - N_A - N_B - N_N$). The number of each type is assumed to be strictly positive.

Whether the insurance firm is chosen by type 3 individuals depends on the level of disutility. That is, their disutility can be written as:

$$Min\left\{P_A + (z-x)^2, P_B + (z-(1-y))^2\right\} \tag{2}$$

where z denotes the ideal point of type 3 individuals and these ideal points are assumed to be distributed uniformly in $[0,1]$. Also assume that the length of the line market is one. Two insurance firms have their varieties at a distance $x, y < 1/2$ from the ends of the line. Also assume that x and y are not only positive values but also negative.[7]

Next, consider the profit functions of both insurance firms. For simplicity, the production costs are assumed to be zero. Then, the profit functions are:

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[7] Negative values of x and y are admitted for the following reason. D'Aspremont et al. (1979) proved the both equilibrium varieties are both ends (corner solutions) in the line market when the disutility function of variety is a quadratic disutility function. However, if there are no ends in the line market, the equilibrium varieties take a negative value (Lambertini, 1994). Moreover, the results of D'Aspremont el al. (1979) are closely related to the form of the disutility function. If the disutility function is assumed to be $8/5$ degree, the equilibrium varieties are not both ends (inner solutions) in the line market (Economides, 1986). Thus, because the characteristics of the equilibrium varieties depend on the length of line market and the disutility function form, we admit negative values in order to avoid the possibility of corner solutions. In summary, negative values merely represent a more differentiated situation.

$$\Pi_A = P_A\left(N_A + N_N D_A\right) \tag{3}$$

$$\Pi_B = P_B\left(N_B + N_N D_B\right) - C\left(\Delta N_B\right) \tag{4}$$

where ΔN_B represents the incremental number of type 2 individuals. Insurance firm B can change individuals from type 4 to type 2 using sales promotion.[8] Because only insurance firm B undertakes sales promotion, $\Delta N_A = 0$ and so $N_A = N_A{}^e$ and $N_B = N_B{}^e + \Delta N_B$ (superscript '*e*' means the initial number of each individual). For simplicity, we assume that the number of type 4 individuals is sufficiently large relative to ΔN_B . $C(\bullet)$ denotes the sales promotion cost function. We assume that $C(0) = 0$, $C'(\bullet) > 0$ and $C''(\bullet) > 0$. D_A and D_B are each insurance firm's market share of type 3 individuals, respectively. In order to derive each market share, the ideal point of the marginal individual whose utility levels are indifferent between two insurance firms, is denoted by $\hat{z}$, shown as follows.[9]

$$P_A + \left(\hat{z} - x\right)^2 = P_B + \left(\hat{z} - (1 - y)\right)^2 \tag{5}$$

Solving the equation (5), we find:

$$\hat{z} = \frac{1 + x - y}{2} + \frac{P_B - P_A}{2(1 - x - y)}. \tag{6}$$

Because the ideal point is distributed uniformly in $[0,1]$, each market share is:

$$D_A = \hat{z} = \frac{1 + x - y}{2} + \frac{P_B - P_A}{2(1 - x - y)}, \tag{7}$$

$$D_B = 1 - \hat{z} = \frac{1 - x + y}{2} - \frac{P_B - P_A}{2(1 - x - y)}. \tag{8}$$

3.2. Deriving the equilibrium in the second and third stages

In this section, we derive the equilibrium in the second and third stages before deriving the subgame perfect equilibrium that is the common equilibrium concept solving multistage (perfect information) game. This equilibrium is obtained by

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[8] Some studies discuss the sales promotion in the spatial competition model. For example, Economides (1989) uses sales promotion to reduce the disutility arising from differences between individual's ideal and actual varieties. Bloch and Manceau (1999) build a model where sales promotion can change the form of the variety distribution function. Our interpretation of sales promotion differs from these previous studies. The justification of our interpretation is that individuals consider sales promotion not as a complement of the insurance product, rather as another service in addition to the insurance product (Crosby and Stephens, 1987).

[9] As long as the disutility function is a convex function, there is a marginal individual in the range $[x, 1 - y]$. For details, see Economides (1984).

backward induction. Thus, we solve the game by analyzing second and third stages, given the decision taken in the previous stages.

Third stage:

The profit maximization conditions can be written as:

$$\frac{\partial \Pi_A}{\partial P_A} = N_A + N_N D_A - \frac{P_A N_N}{2(1-x-y)} = 0, \quad (9)$$

$$\frac{\partial \Pi_B}{\partial P_B} = N_B + N_N D_B - \frac{P_B N_N}{2(1-x-y)} = 0. \quad (10)$$

From the equations (9) and (10), each best-response function is:

$$P_A = \frac{(1-x-y)N_A}{N_N} + \frac{(1+x-y)(1-x-y)}{2} + \frac{P_B^*}{2}, \quad (11)$$

$$P_B = \frac{(1-x-y)N_B}{N_N} + \frac{(1-x+y)(1-x-y)}{2} + \frac{P_A^*}{2} \quad (12)$$

where '*' represents the equilibrium value. Solving equations (11) and (12) yields:

$$P_A^* = \frac{2(1-x-y)(2N_A + N_B)}{3N_N} + \frac{(3+x-y)(1-x-y)}{3}, \quad (13)$$

$$P_B^* = \frac{2(1-x-y)(N_A + 2N_B)}{3N_N} + \frac{(3-x+y)(1-x-y)}{3}. \quad (14)$$

Second stage:

The equilibrium in the second stage can be expressed as the varieties that are derived by maximizing profit given equations (13) and (14). Each profit function (3) and (4) transforms to the reduced form as follows:

$$\Pi_A = P_A^*(x,y)\left(N_A + N_N D_A\left(x, y, P_A^*(x,y), P_B^*(x,y)\right)\right), \quad (15)$$

$$\Pi_B = P_B^*(x,y)\left(N_B + N_N D_B\left(x, y, P_A^*(x,y), P_B^*(x,y)\right)\right) - C(\Delta N_B) \quad (16)$$

where $P_A^*(x,y)$ and $P_B^*(x,y)$ represent the equilibrium prices denoted by the equations (13) and (14) as a function of both varieties. In this stage, it is necessary to calculate $d\Pi_A/dx$ ($d\Pi_B/dy$) instead of $\partial\Pi_A/\partial x$ ($\partial\Pi_B/\partial y$) because the change in x has not only a direct effect on profits but also an indirect effect on profits through its change on equilibrium prices. Thus, the profit maximization condition of insurance firm A can be shown as:

$$\frac{d\Pi_A}{dx} = \frac{\partial \Pi_A}{\partial P_A} \bullet \frac{\partial P_A^{\ *}}{\partial x} + P_A^{\ *}\left(\frac{\partial D_A}{\partial x} + \frac{\partial D_A}{\partial P_B} \bullet \frac{\partial P_B^{\ *}}{\partial x}\right) = 0\,. \tag{17}$$

From equation (9) and $P_A^{\ *} > 0$, then:

$$\frac{d\Pi_A}{dx} = \frac{\partial D_A}{\partial x} + \frac{\partial D_A}{\partial P_B} \bullet \frac{\partial P_B^{\ *}}{\partial x} = 0\,. \tag{18}$$

In accordance with Fudenberg and Tirole (1984) and Tirole (1988), hereafter the first term of the equation (18) ($\partial D_A / \partial x$) is denoted the 'direct effect', while the second term of the equation (18) ($(\partial D_A / \partial P_B)(\partial P_B^{\ *} / \partial x)$) is referred to as the 'strategic effect'. The former relates to the degree of market share and the latter relates to the degree of price competition. From equations (7), (13), and (14), the difference between the two equilibrium prices is represented as:

$$P_B^{\ *} - P_A^{\ *} = \frac{2(1-x-y)(-N_A+N_B)}{3N_N} + \frac{2(1-x-y)(y-x)}{3}\,. \tag{19}$$

Each derivative in equation (18) can then be computed as follows.

$$\frac{\partial D_A}{\partial x} = \frac{1}{2} + \frac{P_B^{\ *} - P_A^{\ *}}{2(1-x-y)^2} = \frac{3-5x-y^*}{6(1-x-y^*)} + \frac{-N_A+N_B}{3N_N(1-x-y^*)} \tag{20}$$

$$\frac{\partial D_A}{\partial P_B} = \frac{1}{2(1-x-y^*)} \tag{21}$$

$$\frac{\partial P_B^{\ *}}{\partial x} = \frac{2}{3}\left(-2 + x - \frac{N_A + 2N_B}{N_N}\right) \tag{22}$$

Substituting equations (20) to (22) into equation (18), we show:

$$\frac{d\Pi_A}{dx} = \frac{3-5x-y^*}{6(1-x-y^*)} + \frac{-N_A+N_B}{3N_N(1-x-y^*)} + \frac{1}{3(1-x-y^*)} \bullet \left\{-2 + x - \frac{N_A+2N_B}{N_N}\right\} = 0. \tag{23}$$

From the equation (23), the reaction function of insurance firm A is given by:

$$x = -\frac{1+y^*}{3} - \frac{2(2N_A+N_B)}{3N_N}\,. \tag{24}$$

By the same computations, the reaction function of insurance firm B is:

$$y = -\frac{1+x^*}{3} - \frac{2(N_A + 2N_B)}{3N_N}. \qquad (25)$$

From equations (24) and (25), the equilibrium varieties can be derived as:

$$x^* = -\frac{1}{4} + \frac{-5N_A - N_B}{4N_N} < 0, \qquad (26)$$

$$y^* = -\frac{1}{4} + \frac{-N_A - 5N_B}{4N_N} < 0. \qquad (27)$$

It is easy to verify that $|x^*| \neq |y^*|$ if $N_A \neq N_B$.[10]

3.3. The effect of variety regulation

Before computing the first stage, we derive some characteristics about equilibrium prices and varieties using the results of the second and third stages. First, the equilibrium prices, equilibrium market shares, and equilibrium profits are shown as:[11]

$$P_A^* = P_B^* = \frac{3N^2}{2N_N^2}, \qquad (28)$$

$$D_A^* = \frac{1}{2} + \frac{-N_A + N_B}{2N_N}, \qquad (29)$$

$$D_B^* = \frac{1}{2} - \frac{-N_A + N_B}{2N_N}, \qquad (30)$$

$$\Pi_A^* = \frac{3N^2}{2N_N^2} \bullet \left\{ N_A + N_N\left(\frac{1}{2} + \frac{-N_A + N_B}{2N_N}\right)\right\}, \qquad (31)$$

$$\Pi_B^* = \frac{3N^2}{2N_N^2} \bullet \left\{ N_B + N_N\left(\frac{1}{2} - \frac{-N_A + N_B}{2N_N}\right)\right\} - C(\Delta N_B). \qquad (32)$$

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[10] In Lambertini (1994), the equilibrium varieties are $x^* = y^* = -1/4$ because there are only type 3 individuals in the model. Thus, if there are no type 1 and type 2 individuals ($N_A = N_B = 0$), the equilibrium varieties are exactly the same as Lambertini (1994).

[11] In Lambertini (1994), the equilibrium prices are $P_A^* = P_B^* = 3/2$ because there are only type 3 individuals in the model. Thus, it is easy to verify that the equilibrium varieties are exactly the same as Lambertini (1994) if $N_N = N$.

However, some conditions are needed to realize the above equilibrium values because $D_A^*, D_B^* \leq 1$ by definition. From equations (29) and (30),

$$D_A^* \leq 1 \Rightarrow -N_A + N_B \leq N_N, \tag{33}$$

$$D_B^* \leq 1 \Rightarrow N_A - N_B \leq N_N. \tag{34}$$

Thus, both equations (33) and (34) are satisfied only if:

$$|N_A - N_B| \leq N_N. \tag{35}$$

Equation (35) means that there are a relatively large number of type 3 individuals and/or the difference between the number of type 1 and type 2 individuals is relatively small. Hereafter, we assume that the equation (35) is always satisfied.

Let us consider the difference in the equilibrium varieties. From equations (26) and (27), this difference is shown as:

$$y^* - x^* = \frac{N_A - N_B}{N_N}. \tag{36}$$

Then, we have:

$$N_A \underset{=}{\overset{<}{>}} N_B \Leftrightarrow |x^*| \underset{=}{\overset{<}{>}} |y^*|. \tag{37}$$

From the equation (37), we find that the degree of product differentiation defined by the distance from the original point (0 and 1 respectively) is closely related to the magnitude of brand loyalties. The insurance firm with a higher brand loyalty is differentiated more than the rival insurance firm. On the other hand, because $x^*, y^* < 0$ and the individuals' ideal points are uniformly distributed in $[0,1]$, the less differentiated insurance firm is desirable for type 3 individuals. Thus, the insurance firm that has higher brand loyalty obtains a smaller number of type 3 individuals.

Equation (28) indicates that both equilibrium prices are the same regardless of brand loyalty. The reason is that the difference in brand loyalties can completely reflect the differences in the product differentiations. However, it is not certain that this situation will be realized in the actual insurance market because some regulations restrict the degree of product differentiation. In order to discuss the effect of these regulations, consider the case where both equilibrium varieties are the corner solutions 0 and 1 because of variety regulation that restricts the variety range. Furthermore, in order to distinguish the unregulated case, the equilibrium prices and varieties in the regulated case are represented as P_A^{C*}, P_B^{C*}, x^{C*}, and y^{C*}, respectively. Unlike the unregulated case discussed earlier, $x^{C*} = y^{C*} = 0$ is always satisfied through regulation. In this situation, from equation (19) the difference between the two equilibrium prices is shown as:

$$P_B^{C*} - P_A^{C*} = \frac{2(-N_A + N_B)}{3N_N}. \tag{38}$$

Thus, from equation (38), we find the following relation concerning equilibrium prices.

$$N_B \overset{>}{\underset{=}{<}} N_A \Leftrightarrow P_B^{C*} \overset{>}{\underset{=}{<}} P_A^{C*} \tag{39}$$

Asymmetric equilibrium prices are realized when variety regulation exists. It can be interpreted that higher brand loyalty leads to higher prices instead of a greater degree of differentiation.

In order to analyze the above point more generally, the line market is depicted not as a straight line but as a segment because of the existence of variety regulation. Both limits of the segment are located in 1/2 symmetry. We denote $r \geq 1$ as the length of the segment. In order to simplify the notation, we also define $R \equiv (r-1)/2$. Further, we denote $\Psi \equiv P_B^{C*} - P_A^{C*}$. In the case of $N_B > N_A$, the function Ψ can be divided into three intervals. In the first interval, R is from 0 to $|x^*|$. In this interval, both varieties become corner solutions, that is, $|x^*| = |y^*|$ and Ψ increases proportional to R. In the second interval, R is from $|x^*|$ to $|y^*|$. In this interval, insurance firm A's variety is the inner solution, while insurance firm B's variety is the corner solution. Thus, the equilibrium varieties are asymmetric, that is $|x^*| < |y^*|$ because the insurance firm A does not have the incentive to increase the degree of differentiation, even if the length of the segment becomes longer. Also, because $\partial\Psi/\partial y > 0$ and $\partial^2\Psi/\partial y^2 < 0$, function Ψ forms a decreasing and gradually increasing function of R.[12] In the third interval, R is more than $|y^*|$. In this interval, both varieties become inner solutions. This is the same as the unregulated case discussed earlier. From equation (28), both equilibrium prices are always same and so $\Psi = 0$ regardless of R. As a result, we can depict function Ψ as Figure 1 in (R, Ψ) space in the case of $N_B > N_A$.

[12] $\partial\Psi/\partial y = (-2/3)(2y^{C*} + (-N_A + N_B - N_N)/N_N) > 0$ and $\partial^2\Psi/\partial y^2 = -4/3 < 0$.

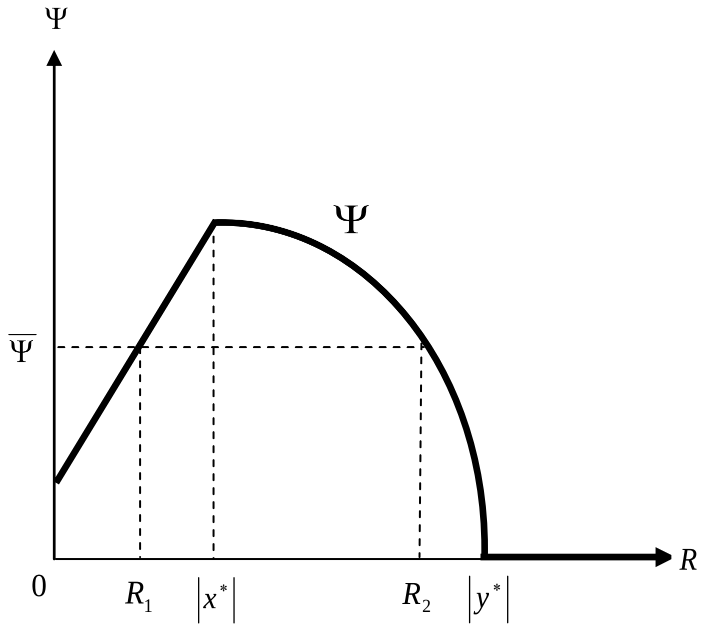

Figure 1: The Function Form of Ψ

Figure 1 also indicates that the difference between the equilibrium prices does not necessarily show the degree of differentiation. For example, consider any equilibrium price difference $\overline{\Psi}$ in Figure 1. It is clear that there are two cases to realize $\overline{\Psi}$ in Figure 1. In the case of R_1, the equilibrium varieties are symmetric. In contrast, in the case of R_2, they are asymmetric.

Finally, the relationship between R and the equilibrium varieties and prices can be generally represented as follows.

If $R \leq Min\{|x^*|,|y^*|\}$, then the equilibrium varieties are symmetric while the equilibrium prices are asymmetric.

If $Min\{|x^*|,|y^*|\} < R < Max\{|x^*|,|y^*|\}$, then both the equilibrium varieties and prices are asymmetric.

If $Max\{|x^*|,|y^*|\} \leq R$, then the equilibrium varieties are asymmetric while the equilibrium prices are symmetric.

Thus, there is no possibility of realizing the equilibrium where both varieties and prices are symmetric, regardless of the length of the segment.[13] In contrast, if there are

[13] There are some studies that derive asymmetric equilibriums in the spatial competition model. Polo (1991) discussed a case where some individuals' price information is imperfect and where there may be asymmetric price equilibrium. Tabuchi and Thisse (1995) introduced the symmetric triangular density function of individuals' ideal varieties producing an asymmetric equilibrium with variety and price. Santore (1999) analyzed a case where some individuals are uninterested in varieties at all and also indicated asymmetric price equilibrium.

no type 1 and type 2 individuals, the degree of differentiation and prices in the equilibrium are always the same, regardless of the existence of regulation.

3.4. The optimal level of sales promotion

The reduced profit function of insurance firm B in the first stage can be written as follows.

$$\Pi_B = P_B^*\left(x^*(\Delta N_B), y^*(\Delta N_B), \Delta N_B\right)$$
$$\times\left(N_B + N_N D_B^*\left(x^*(\Delta N_B), y^*(\Delta N_B), P_A^*\left(x^*(\Delta N_B), y^*(\Delta N_B), \Delta N_B\right), P_B^*\left(x^*(\Delta N_B), y^*(\Delta N_B), \Delta N_B\right)\right)\right)$$
$$-C(\Delta N_B) \quad (40)$$

$x^*(\Delta N_B)$, $y^*(\Delta N_B)$, $P_A^*\left(x^*(\Delta N_B), y^*(\Delta N_B), \Delta N_B\right)$ and $P_A^*\left(x^*(\Delta N_B), y^*(\Delta N_B), \Delta N_B\right)$ represent the equilibrium varieties and prices in the form of the function of sales promotion.

As in the second stage, it is necessary to calculate not $\partial\Pi_B/\partial\Delta N_B$ but $d\Pi_B/d\Delta N_B$ because the change in ΔN_B has not only a direct effect on profits but also an indirect effect on profits through its effect on equilibrium varieties and prices. The profit maximizing condition with respect to ΔN_B is then:

$$\frac{d\Pi_B}{d\Delta N_B} = \frac{\partial\Pi_B}{\partial\Delta N_B} + \frac{dx^*}{d\Delta N_B}\bullet\frac{d\Pi_B}{dx} + \frac{dy^*}{d\Delta N_B}\bullet\frac{d\Pi_B}{dy}$$
$$+\frac{\partial P_A^*}{\partial\Delta N_B}\bullet\frac{\partial\Pi_B}{\partial P_A} + \frac{\partial P_B^*}{\partial\Delta N_B}\bullet\frac{\partial\Pi_B}{\partial P_B} = 0. \quad (41)$$

Because $d\Pi_B/dy = 0$ and $\partial\Pi_B/\partial P_B = 0$, equation (41) can be rewritten as:

$$\frac{d\Pi_B}{d\Delta N_B} = \frac{\partial\Pi_B}{\partial\Delta N_B} + \frac{dx^*}{d\Delta N_B}\bullet\frac{d\Pi_B}{dx} + \frac{\partial P_A^*}{\partial\Delta N_B}\bullet\frac{\partial\Pi_B}{\partial P_A} = 0. \quad (42)$$

Thus, the equilibrium sales promotion level is satisfied with equation (42). However, equation (42) is the equilibrium condition without any regulation. If there is variety regulation, the equilibrium condition for sales promotion may change. If so, does the existence of the regulation raise the sales promotion level? In order to confirm this, consider the situation where there is regulation that restricts the variety range as discussed in the previous section. In this situation, the degree of differentiation cannot be expanded and so the term of $\left(dx^*/d\Delta N_B\right)$ in the equation (42) is equal to be zero. Thus, the effect of variety regulation on the level of sales promotion can be expressed by $\left(dx^*/d\Delta N_B\right)\left(d\Pi_B/dx\right)$ in equation (42). If $\left(dx^*/d\Delta N_B\right)\left(d\Pi_B/dx\right)$ in equation (42) is positive, this means that the presence of variety regulation results in a lower level of sales promotion.

From equation (26), it is easy to verify that $dx^*/d\Delta N_B < 0$. In contrast, $d\Pi_B/dx$ can be written in reference to the explanation in the second stage,

$$\frac{d\Pi_B}{dx} = \frac{\partial \Pi_B}{\partial P_B} \bullet \frac{\partial P_B^*}{\partial x} + P_B^* \left(\frac{\partial D_B^*}{\partial x} + \frac{\partial D_B^*}{\partial P_A} \bullet \frac{\partial P_A^*}{\partial x} \right). \tag{43}$$

Also, because $\partial \Pi_B / \partial P_B = 0$ and $P_B^* > 0$, then:

$$Sign\left[\frac{d\Pi_B}{dx} \right] = Sign\left[\frac{\partial D_B^*}{\partial x} + \frac{\partial D_B^*}{\partial P_A} \bullet \frac{\partial P_A^*}{\partial x} \right] \tag{44}$$

where $Sign[\bullet]$ denotes the operator of the equation's sign.

Each derivative on the left-hand side of equation (44) is calculated as:

$$\frac{\partial D_B^*}{\partial x} = -\frac{1}{2} < 0, \tag{45}$$

$$\frac{\partial D_B^*}{\partial P_A} = \frac{N_N}{3N} > 0, \tag{46}$$

$$\frac{\partial P_A^*}{\partial x} = -\frac{N}{2N_N} < 0. \tag{47}$$

Substituting equations (45) to (47) into equation (44), we confirm:

$$Sign\left[\frac{d\Pi_B}{dx} \right] = (-) + (+)(-) < 0. \tag{48}$$

In the end, $(dx^*/d\Delta N_B)(d\Pi_B/dx)$ in equation (42) is positive so that regulation lowers the level of sales promotions.

Next, consider whether lowering the level of sales promotion is socially desirable. More particularly, we compare the sales promotion levels in the cases of regulated and unregulated insurance markets.

The price level can be ignored when comparing social surplus. Thus, we only analyze the variety and sales promotion aspects. Since regulation lowers the degree of differentiation, type 3 individuals' surpluses are increased. Also, the producer surplus is increased because sales promotion costs are lowered by introducing variety regulation. In contrast, lowering the sales promotion level decreases the number of individuals shifting from type 4 to type 2. Thus, producer surplus lowers u_B per individual.

In general, we cannot find a unique result concerning the effect of introducing variety regulation. Roughly speaking, the regulation increases the social surplus when the reservation price is not too high. In other words, there is a certain rationale in introducing variety regulation as an attempt to prevent excess product differentiation and sales promotion.

4. Concluding remarks

This article analyzes the insurance market with horizontal differentiation and brand loyalties using a spatial competition model. We build a three-stage game that includes sales promotion (the first stage), varieties (the second stage), and premiums (the third stage). The following results are obtained.

1. The insurance firm that has higher brand loyalty realizes a higher degree of differentiation and sets higher premiums than its rivals. This can explain why domestic insurance firms with relatively higher brand loyalty have a higher degree of differentiation and set higher premiums than foreign insurance firms.
2. Introducing variety regulation leads to a lower degree of differentiation and level of sales promotion. Such regulation may be desirable in terms of social surplus. This can explain why variety regulation for domestic insurance is more rigid than for business insurance because there are many individuals who have brand loyalty in the domestic insurance market, and so the disadvantages of introducing variety regulation appear small.

However, our analysis has some limitations. First, we did not consider the multiple variety and/or quality (multidimensional) cases.[14] Second, we analyzed a static model. It would be more appropriate to construct a dynamic model to analyze insurance contracts that are likely to be long-term. Third, if both insurance firms choose a sales promotion level, the spillover effect has to be considered. Roughly speaking, both insurance firms may wish to choose a lower investment level than in our model.

We believe the results obtained in this article are of considerable help in explaining horizontal differentiation in the insurance market. However, in order to enhance the contribution of this research, it is applicable to the actual insurance market needs to be studied in the future.

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[14] For the analysis of multidimensional spatial competition, see Irmen and Thisse (1998).

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